# VEERMATA JIJABAI TECHNOLOGICAL INSTITUTE

[Central Technological Institute, Maharashtra State]

Matunga, Mumbai-400 019

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| SEMESTER EXAMINATION | *May - 2012* | DATE OF EXAM: | *07-05-2012* |
| SEMESTER & PROGRAM: IV | *S.Y.B.Tech (Comp/I.T/Elect/Etrx/Extc)* | TIME | *1.30 p.m.to4.30p.m.* |
| TIME ALLOWED | *3 HRS.* | MARKS | *100* |
| COURSE (CourseCode) : | *(MA0206) Engineering Mathematics-II* | | |

Instructions

1. All the questions are compulsory.
2. All questions carry equal marks.
3. Figures to the right indicate full marks.
4. Answer to the individual questions must be grouped and written together

Q.1.(a) Evaluate $\int_1^i z^2\,dz$ along the circle $|z| = 1$. (4)

(b) i) What is the relation between trace and Eigen values of a matrix ?

ii) If A is nonsingular of order Three having its Characteristic roots in A.P. If Trace of $A = 15, |A| = 80$, Find Trace of $A^{-1}$ . (4)

(c) Expand the function $f(z) = \frac{1}{z}$ about $z = 2$ in Taylor's series. Obtain its radius of Convergence. (4)

(d) Compute $\int_c \bar{F}.d\bar{r}$ ,where $\bar{F} = \frac{iy-jx}{x^2+y^2}$ and $c$ is the circle $x^2 + y^2 = 1$ traversed counter clockwise. (4)

(e) Define : i) Isolated Singularity ii) Removable Singularity iii) Essential Singularity iv) Pole of order m. (4)

Q.2 . (a) Evaluate the integral by using the residue theorem

i) $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}dx$ ii ) $\int_{-\infty}^{\infty} \frac{cosx}{(x^2+1)(x^2+4)}dx.$ (8)

(b) Find the Eigen values and Eigen vectors of

$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. Determine whether the Eigen vectors are orthogonal. (6)

(c) Using Gauss's Divergence theorem evaluate $\iint_s (ax^2 + by^2 + cz^2)\,ds$ over the sphere $x^2 + y^2 + z^2 = 1.$ (6)

Q.3. (a) Verify Green's theorem for $\oint_c (x^2 - 2xy)dx + (x^2y + 3)\,dy$ where $c$ is boundary of the region defined by $y^2 = 8x$ and $x = 2$ . (8)

(b) Evaluate $\oint_c \frac{2z^2+5}{(z+2)^3(z^2+4)z}dz$ where $c$ is the circle $|z - 2i| = 6$ (6)

(c) State the cayley-Hamilton theorem and determine $A^{-1}, A^{-2}, A^{-3}$ if

$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$ (6)

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Q.4.(a) Verify Stoke's theorem for $\bar{F} = (x^2 + y - 4)i + 3xyj + (2xz + z^2)k$ over the surface of hemisphere $x^2 + y^2 + z^2 = 16$ above the xy-plane. (8)

(b) Find the Taylor's and Laurent's series of the function $f(z) = \frac{1}{z^2-3z+2}$ in the region
(i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$ iv) $0 < |z-1| < 1$. (6)

(c) Show that the matrix $A = \begin{bmatrix} 1 & -4 & -4 \\ 0 & 4 & 4 \\ 0 & -6 & -3 \end{bmatrix}$ is similar to a diagonal matrix. Also find transforming matrix and diagonal form . (6)

**OR**

(c) If $\bar{r}$ is the positive vector of the point $(x, y, z)$ and $r$is the modulus of $\bar{r}$ then prove that $r^n\bar{r}$is an irrotational vector for any value of n buts solenoidal only if $n = -3$, (6)

Q.5.(a) Reduce the following form to canonical form and find rank, index and signature
$Q.F = x_1^2 + 6x_2^2 + 18x_3^2 + 4x_1x_2 + 8x_1x_3 - 4x_2x_3$. (8)

(b) (c) State Cauchy's Integral Formula & prove that $\oint_C \frac{f(z)}{(z-a)} dz = 2\pi i f(a)$ and deduce that $\oint_C \frac{f(z)}{(z-a)^{n+1}} dz = \frac{2\pi i f^n(a)}{n!}$ . (6)

(c) Find the constant $a$ and $b$ so that the surface $ax^2 - byz = (a + 2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4\ at\ (1, -1, 2)$. (6)

**OR**

(c) If $A = \begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$ ,Prove that $A^{50} = \begin{bmatrix} -149 & -150 \\ 150 & 151 \end{bmatrix}$ (6)

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