VEERMATA JIJABAI TECHNOLOGICAL INSTITUTE

[Central Technological Institute, Maharashtra State]

Matunga, Mumbai-400 019

SEMESTER EXAMINATION	May 2013	DATE OF EXAM	
SEMESTER & PROGRAM	*SYBTECH-IV*(Com, IT, El, ETC,EXTC)	TIME	
TIME ALLOWED	*3 HRS.*	MARKS	*100*
COURSE (Course Code) :	*Eng Mathematics(MA0206)*		

Instructions
1. All questions carry equal marks.
2. Figures to the right indicate full marks.

Q1. a If sum of the eigen values of a 3x3 matrix is 6 and product of the eigen value is also 6. If one of the eigen values is one, find the other two eigen values. 4

b Prove that $\bar{F} = \frac{\bar{r}}{r^3}$ is solenoidal. 4

c By using Cauchy's integral formula evaluate : $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$, where "C" is the circle $|z| = 3$ 4

d Find Taylor's series expansion of $f(z) = \frac{1}{4-3z}$ about $z_0 = 1+i$. Determine the region of convergence. 4

e Prove that any two eigen vectors corresponding to two distinct eigen values of a unitary matrix are orthogonal. 4

Q2 a Show that the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ is diagonisable. Find transforming matrix & diagonal matrix. 6

b Using residues, evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)^2(x^2+2x+2)} dx$ 6

c Show that $\bar{F} = (2xyz^2)i+(x^2z^2 + z\cos yz)j + (2x^2yz + y\cos yz)k$ is conservative. Find scalar potential φ such that $\bar{F} = \nabla\varphi$ and hence find the work done by $\bar{F}$ in displacing particle from A(0,0,1) to $B(1, \frac{\pi}{4}, 2)$ along the straight line AB. 8

3 a Using Green's theorem evaluate $\int_c \bar{F}.d\bar{r}$ where "C" is the curve enclosing the region bounded by $y^2 = 4ax$, $x = a$ in the plane $z = 0$ and $\bar{F} = (2x^2y + 3z^2)i+(x^2 + 4yz)j + (2y^2 + 6xz)k$ 6

b Expand $f(z) = \frac{3z-3}{(2z-1)(z-2)}$ in a laurent's series about z =1 convergent in $\frac{1}{z} < |z-1| < 1$. 6

c Show that the form $x_1{}^2 + 2x_2{}^2 + 3x_3{}^2 + 2x_2x_3 + 2x_1x_2 - 2x_3x_1$ is indefinite and find sets of values of x_1, x_2, x_3 which will make the form positive and negative. 8

4 a Find directional derivative of $f(x,y,z)= x^2y^2z^2$ at the point (1,1,-1) in the direction of tangent to the curve $x = e^t$, $y = 2\sin t + 1$, $z =$ t-cost t at t=0. 6

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b If $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$, find the characteristic roots and characteristic vectors of $A^3 + I$. 6

c Find f(z), iff $f(z) = \frac{\phi(z)}{\varphi(z)}$ where $\phi(z)$ & $\varphi(z)$ are complex polynomials of degree 2 has 8

i) pole of order 2 at $z = 1$

ii) residue at z=1 is -1

iii) $f(0) = f(-1) = 0$.

(OR)

i) If $\bar{f} = (x + y + 1)i + j - (x + y)k$. Find $\bar{f}$ curl $\bar{f}$.

ii) Prove that $\nabla.(\nabla x \bar{F}) = 0$ where $\bar{F}$ is a vector point function.

5 a Verify Stoke's theorem $\bar{F} = (x - y - z)i + (y - z - x)j + (z - x - y)k$ over the paraboloid $x^2 + y^2 = 4 - z,\ z \geq 0$ 6

b Evaluate $\int_c \frac{2z+3}{z} dz$, where "C" is 6

i) the upper half of the circle |z|=2

ii) the lower half of the circle |z|=2

c Verify Gauss Divergence theorem for $\bar{F} = xi + yj + z^2k$ and "S" is the closed surface bounded by the cone $x^2 + y^2 = z^2$ and the plane z=1 8

(OR)

Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and verify that it is satisfied by A and hence obtain A^{-1}.